

Exercise 45

Find the limit.

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta}$$

Solution

Rewrite the limit in terms of one that is known.

$$\begin{aligned} \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta} &= \lim_{\theta \rightarrow 0} \frac{\frac{\sin \theta}{\theta}}{1 + \frac{\tan \theta}{\theta}} = \lim_{\theta \rightarrow 0} \frac{\frac{\sin \theta}{\theta}}{1 + \frac{\sin \theta}{\theta \cos \theta}} \\ &= \frac{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}}{\lim_{\theta \rightarrow 0} \left(1 + \frac{\sin \theta}{\theta \cos \theta} \right)} \\ &= \frac{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}}{1 + \left(\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \right) \left(\lim_{\theta \rightarrow 0} \frac{1}{\cos \theta} \right)} \\ &= \frac{1}{1 + (1)(1)} \\ &= \frac{1}{2} \end{aligned}$$